THE DEGENERATION OF TURBULENCE IN A LIQUID WITH INTERNAL ROTATION

V. A. Gorodtsov

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Viscous fluids which possess even a small degree of elasticity behave in quite a different way from viscous fluids in non-steady-state flow which is fairly fast. The presence of elasticity leads to a change in the nature of the damping of small scale (high frequency) turbulent eddies when the turbulence becomes degenerate [1]. The difference in the specific properties of viscoelastic fluids manifests itself in the behavior of these eddies.

The presence of sufficiently large-scale lasting super-molecular formations in the fluid can impart to it "elastic" properties. The fact that the additive lags behind the solvent which is moving and accelerating is associated with the relaxation of the translational degrees of freedom of the composite model. The effect of this relaxation on the damping of high frequency eddies was treated in [1]. In what follows the effect of rotational relaxation is examined. In a fluid with internal circulation the eddies break up in the following manner according to the nature of the damping: the eddies with the largest and smallest scales experience viscous (diffusive) damping, but local relaxation is more important for the medium sized eddies, and they are damped as $\exp(-t/\Theta)$, where θ is a constant proportional to the time of the rotational relaxation.

1. The processes associated with the reorganization of fluid structure, which lead to the appearance of elastic properties are extremely varied. The phenomenological models which describe the flow of such fluids are also very diverse. We may thus expect that there exist qualitatively different types of turbulence for viscoelastic fluids. Although the nonlinear characteristics of the different fluids are evened out in the final stage of the damping of turbulence, the behavior of the motion of the different scales is of interest in the linear stage. In paper [1] it was shown that the degeneration of turbulence for viscoelastic fluids with the same relaxation time has a very general character: the large scale motions are damped in the same way as in a viscous fluid, and the small scale eddies degenerate in a universal manner (independently of the dimension), but more rapidly than the large scale eddies, and in the final stage, for times which are large, the nature of the turbulence damping is completely determined by the large "viscous" eddies, which leads to the asymptotic law $t^{-5/2}$.

However, for the examples of fluids with relaxation of shear stress and shear velocity already considered in [1] the differences in the damping of the motions of the different scales turn out to be characteristic and important.

The behavior of a fluid with shear velocity relaxation is similar to the behavior of a viscous fluid with an effective viscosity which depends on the scale of the motion under consideration. For motion with a wave number k the effective viscosity has the form

$$\nu_\theta(k) = \nu(1 + 2\nu\theta k^2)^{-1}.$$

Here ν and θ are the constants of kinematic viscosity and relaxation time. It is clear from this expression that the effective viscosity $\nu_\theta(k)$ falls off monotonically as the scale of the motion decreases (as k increases). Now if we compare the damping of turbulent fluid motions where there is stress relaxation and the damping of pulsations in a viscous fluid [1], then an effective viscosity may be introduced which will depend on the scale of the motion as follows:

$$\nu_\theta(k) = [1 - (1 - 4\nu\theta k^2)^{1/2}](2\theta k^2)^{-1}.$$

From this it is clear that in contrast with the previous case the effective viscosity increases for the large scale motions (from ν when $k = 0$ to 2ν when $k = (\nu\theta)^{-1/2}/2$), and $(\nu\theta)^{-1/2}$ even becomes complex for small scale motions with $k > 1/2$ (in this case the energy spectrum of the motions oscillates in the high frequency region with a frequency which increases linearly and asymptotically for large k).

Even on the basis of this treatment of the linear stage of the damping we may suppose that the nature of fully developed turbulence will change radically when the scale $(\nu\theta)^{1/2}$ becomes equal to the scale for which the main part of the turbulent energy in a viscous fluid occurs. If the length $(\nu\theta)^{1/2}$ is comparable to the inner scale of the turbulence this may lead to a rearrangement of the high energy end of the spectrum of turbulent kinetic energy. We may thus expect a marked change in those turbulent motions whose scale is less than or of the same order as the dimension of the viscous zone of influence of the relaxation process $(\nu\theta)^{1/2}$. In what follows the behavior of turbulent motions is considered during the final stages of damping for the case of a fluid with rotational structure relaxation.

2. The numerous investigations which have been made of the hydrodynamics of dilute solutions of polymers indicate that insignificant quantities of polymer additive have a marked effect on the turbulent flow of fluids. According to paper [2], one of the causes of the damping effect of a polymer on turbulent eddies is the presence of large supermolecular polymer aggregates in the solution. The presence of a coarse additive leads to additional dissipative effects. The presence of an additive leads to an effect by which the lagging additive is tuned to the motion of the fluid. The effect of this on fluid turbulence was treated in paper [1] within the framework of a phenomenological model with relaxation of deformation velocities. In addition to this, tuning of the rotational motion of the additive to the turbulent eddies occurs, which leads to vortex relaxation of the fluid and additional dissipation of energy. A phenomenological fluid model which took this effect into account was proposed in [3]. Here we shall treat some characteristic features of turbulence for this model.

The complete system of equations for an incompressible fluid with internal circulation is

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\mathbf{v} = -\nabla(p + \mathbf{M}\boldsymbol{\Omega} - \alpha \mathbf{M}^2) + (\eta + {}^1/_4\gamma)\Delta\mathbf{v} + {}^1/_2\alpha\gamma \operatorname{rot}\mathbf{M},$$

$$\frac{\partial \mathbf{M}}{\partial t} + (\mathbf{v}\nabla)\mathbf{M} = \gamma(\boldsymbol{\Omega} - \alpha\mathbf{M}) + \mu\Delta\mathbf{M}, \qquad (\nabla\mathbf{v}) = 0\,. \quad (2.1)$$

Here η, γ, μ, and $\alpha\,(\geq 0)$ are the coefficients of the first and third (vortex) viscosities, the diffusion of internal momentum **M** and the structure coefficient; v is the velocity vector, p is the pressure. The system of units is chosen so that the density $\rho = 1$, i.e., all quantities have been made kinematic (only length and time enter into the dimensionality) by dividing them by the constant density.

Taking the curl of the first equation of (2.1), we obtain

$$\frac{\partial \boldsymbol{\Omega}}{\partial t} - \operatorname{rot}[\mathbf{v}\boldsymbol{\Omega}] = (\eta + {}^1/_4\gamma)\Delta\boldsymbol{\Omega} + {}^1/_4\alpha\gamma \operatorname{rot}\operatorname{rot}\mathbf{M},$$

$$\frac{\partial \mathbf{M}}{\partial t} + (\mathbf{v}\nabla)\mathbf{M} = \mu\Delta\mathbf{M} + \gamma(\boldsymbol{\Omega} - \alpha\mathbf{M}),$$

$$\boldsymbol{\Omega} = {}^1/_2 \operatorname{rot}\mathbf{v}, \qquad (\nabla\mathbf{v}) = 0\,. \quad (2.2)$$

The system of equations (2.2) is like a similar system of equations in magnetohydrodynamics; however, here there can be no question of freezing-in of the field **M**. On the other hand the field M exhibits both local departures from the equilibrium state $\alpha^{-1}\Omega$, as well as diffusive nonequilibrium for a nonuniform distribution.

Equations (2.2) may be very much simplified in the case of flow which allows linearization (for example in the final stage of damping turbulence). In this case when the field **M** is eliminated from the first two equations of (2.2) we obtain

$$\frac{\partial^2 \boldsymbol{\Omega}}{\partial t^2} + [\alpha\gamma - (\eta + {}^1/_4\gamma + \mu)\Delta]\frac{\partial \boldsymbol{\Omega}}{\partial t} - \alpha\gamma\eta\Delta\boldsymbol{\Omega} + \mu(\eta + {}^1/_4\gamma)\Delta^2\boldsymbol{\Omega} = 0\,,$$

$$\gamma\boldsymbol{\Omega} = \left(\frac{\partial}{\partial t} - \mu\Delta + \alpha\gamma\right)\mathbf{M}\,. \quad (2.3)$$

It is clear from (2.3) that the variation of angular velocity Ω and internal momentum **M** for the motion of a given scale size occurs in the same way in the final stage, although the initial distributions may have been different (since the equations do not coincide for the nonlinear stage), and consequently it suffices to speak about one of the fields only.

The degeneration of motions with various scale sizes occurs independently in the final stage, and so it is convenient to describe it in terms of wave space, taking the Fourier transform of Eq. (2.3). When this is done the equation for

$$\boldsymbol{\omega}(\mathbf{k}) = \frac{1}{8\pi^3}\int \boldsymbol{\Omega}(\mathbf{r})\, e^{-i\mathbf{k}\mathbf{r}}\, d^3r$$

assumes the form

$$\frac{\partial^2 \boldsymbol{\omega}}{\partial t^2} + [\alpha\gamma + (\eta + {}^1/_4\gamma + \mu)k^2]\frac{\partial \boldsymbol{\omega}}{\partial t} + k^2\alpha\gamma\eta\boldsymbol{\omega} + \mu(\eta + {}^1/_4\gamma)k^4\boldsymbol{\omega} = 0\,. \quad (2.4)$$

If we look for a solution in the form e^{st} we obtain for s

$$2s = -[\alpha\gamma + k^2(\eta + {}^1/_4\gamma + \mu)] \pm \{[\alpha\gamma + k^2(\eta + {}^1/_4\gamma + \mu)]^2 - 4\alpha\gamma\eta k^2 - \mu(\eta + {}^1/_4\gamma)k^4\}^{1/2}. \quad (2.5)$$

Since this expression is always real and negative, motions of all scale sizes are damped smoothly (without oscillation). Of the two roots in (2.5) it suffices to retain the one which gives rise to the slower damping, i.e., that with the plus sign. Expanding the expressions under the radical in series and introducing the symbol $\tau = (\alpha\gamma)^{-1}$ for the time of local relaxation [3] of internal momentum **M**, we obtain

$$s(k) = -\eta k^2 \frac{1 + \tau k^2\mu(1 + {}^1/_4\gamma/\eta)}{1 + \tau k^2(\eta + \mu + {}^1/_4\gamma)}\left\{1 + \tau\eta k^2 \frac{1 + \tau k^2\mu(1 + {}^1/_4\gamma/\eta)}{[1 + \tau k^2(\eta + \mu + {}^1/_4\gamma)]^2} + \ldots\right\}. \quad (2.6)$$

This expression (compare with formula (2.3) of paper [1]) shows that the damping of motions in such a fluid may be described by an effective viscosity $k^{-2}s(k)$, which tends to η as $k \to 0$ (large scale irregularities), i.e., the large eddies are damped with time as $\exp(-\eta \cdot k^2 t)$ in the same way as eddies in an ordinary viscous fluid with viscosity η.

We again make use of Eqs. (2.3) in order to describe pulsations with large wave numbers k: it is clear that the field is specified by the change of the field **M**, which occurs in different ways depending on which relaxation mechanism predominates: local relaxation with time τ or diffusive relaxation with a characteristic time $(k^2\mu)^{-1}$ for motion having a scale $\sim k^{-1}$.

For motions whose wave numbers satisfy the condition $\mu k^2 \gg 1/\tau$ (small scale motions), diffusive relaxation determines the basic change of the field **M**. In this case it may be seen from (2.3), that damping of the motions is described by two types of relations: $\exp[-(\eta + \gamma/4)k^2 t]$ and $\exp[-\mu k^2 t]$. When one of them predominates, the damping is similar to the damping of motion in a viscous fluid with viscosity $\eta + \gamma/4 < \mu$ or with viscosity $\mu < \eta + \gamma/4$.

In the opposite case $\mu k^2 \ll 1/\tau$, when local relaxation of the vector **M** exerts the predominant influence, Eqs. (2.3) assume the form

$$\frac{\partial^2 \boldsymbol{\Omega}}{\partial t^2} + \left[\frac{1}{\tau} - (\eta + {}^1/_4\gamma)\Delta\right]\frac{\partial \boldsymbol{\Omega}}{\partial t} - \frac{\eta\Delta\boldsymbol{\Omega}}{\tau} = 0,$$

$$\gamma\boldsymbol{\Omega} = \left(\frac{\partial}{\partial t} + \frac{1}{\tau}\right)\mathbf{M}\,. \quad (2.7)$$

It is clear from this that for $k^2(\eta + \gamma/4) \ll 1/\tau$ (large scale motions) the medium rapidly approaches local equilibrium and the damping has a purely viscous character. The smallness of k is determined here by the relations $k^2 \ll (\tau\mu)^{-1}$, $k^2 \ll \tau^{-1}(\eta + \gamma/4)^{-1}$. On the other hand, for $k^2(\eta + \gamma/4) \gg 1/\tau$, local relaxation plays the main role and the damping law has the universal form $\exp(-t/\theta)$. Here the relaxation time $\theta = \tau(1 + \gamma/4\eta) > \tau$. Motions with wave numbers k in the interval $(\mu\tau)^{-1} \gg k^2 \gg \tau^{-1}(\eta + \gamma/4)^{-1}$, assuming that this interval exists, are damped in this way.

Thus, generally speaking, wave number space may be divided into three regions: the region of small k where the damping of motion is similar to damping in a viscous fluid with viscosity η, the region of medium k with universal damping $\exp(-t/\theta)$, and finally the region of large k where the damping is like that in a viscous fluid with viscosity constants $\eta + \gamma/4$ and μ. The fact that the damping of motions in a fluid with internal circulation is of a more complicated nature than was encountered in [1] is associated with the presence of a large number of characteristic relaxation times: in addition to the viscous damping times $(\eta k^2)^{-1}$, $k^{-2}(\eta + \gamma/4)^{-1}$, which characterize the rotational rearrangement of the fluid structure.

Because of the linear relation (2.3) between Ω and **M** vortex irregularities of the field **M** with given k are damped with time in the same way as pulsations of Ω, while the nonrotational part of the field is damped as $\exp[-t(1/\tau + \mu k^2)]$. As regards the total kinetic energy of turbulent motion, we note that in the final stage of degeneration of the turbulence, when motions with scale sizes less than the dimension of the zone of influence of structure relaxation $\tau^{1/2}(\eta + \mu + \gamma/4)^{1/2}$ are practically damped, the decrease of kinetic energy is described by the law $t^{-5/2}$, as for a viscous fluid.

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